The Neural Mechanisms of Parallel Individuation and Numerical Approximation

Jean Tang-Lonardo[1], Nick Bisbee[2], Maitri Jain[1], Erin Kirby[1], Sadra Gerami[1], Melissa Coffel[1], Daniela Sezcon-Cepeda[1], Kai Gilchrist[1], Samah Abdelrahim[1], Sungbong Kim[1], and Peter Gordon[1]

(1) Teachers College, Columbia University, (2) Johns Hopkins University, **Contact: pgordon@tc.edu**

Theoretical Overview

Research on numerical cognition using behavioral, neuroimaging, developmental and cross-cultural methods, converges on the conclusion that there are two distinct systems for the perception of numerical quantity:

1. A small-number system (1~3) invoking subitizing and object-tracking in "parallel individuation" (Gordon, 1994).
2. A large-number system (4+) that is based on Weberian analog magnitude estimation/numerical approximation.

Previous EEG research in numerical cognition:

- Previous studies have found ERP N1 negativities associated with perception of numerical values in the posterior Parietal-Occipital-Temporal (POT) region (Temple & Posner, 1998).
- Hyde & Spelke (2012) employed a passive numerical viewing task to examine ERPs associated with changes within the small number (1-2-3) and large number (8-16-24) range. Participants viewed 4 dot displays with the same number of dots (adaptation), followed by a test array with either the same number or a small vs. large change. No changes crossed between small and large number categories.
- Hyde and Spelke (2012) examined test items that involved changes (or no change) following the adaptation phase. Within the small number range (1~3), the N1 ERP showed scaling of magnitude with numerical quantity.
- A later positivity discriminated ratio changes: P2p for large numbers (8-16-24); P3 for small numbers (1-2-3).
- Rationale for electrode analysis choices was based upon the tendency to observe larger P3b amplitudes in the parietal areas during change processes (Hyde & Spelke, 2012; Polich, 2011).

Study Design

<u>**Adapted Design from Hyde & Spelke (2012):**</u>

- Our study (***Fig. 1***) involved a sequential presentation of dot arrays in "Small" (1, 2, 3) and "Large" (4, 5, 6) numerosities to participants.
- **Stimulus duration:** 250 ms
- **Interstimulus interval:** 750 - 1250 ms
- In each trial, to prime/habituate the brain to a numerical value, the same value was presented for three to five slides.
- Followed by a **Target** slide from either of these conditions:
  - **a) No Change** (Same number)
  - **b) Change within Small or Large** numbers (e.g., 1→3; 2→1; 6→5)
  - **c) Cross-over between Small and Large** numbers (e.g., 3→4; 2→5; 6→3)

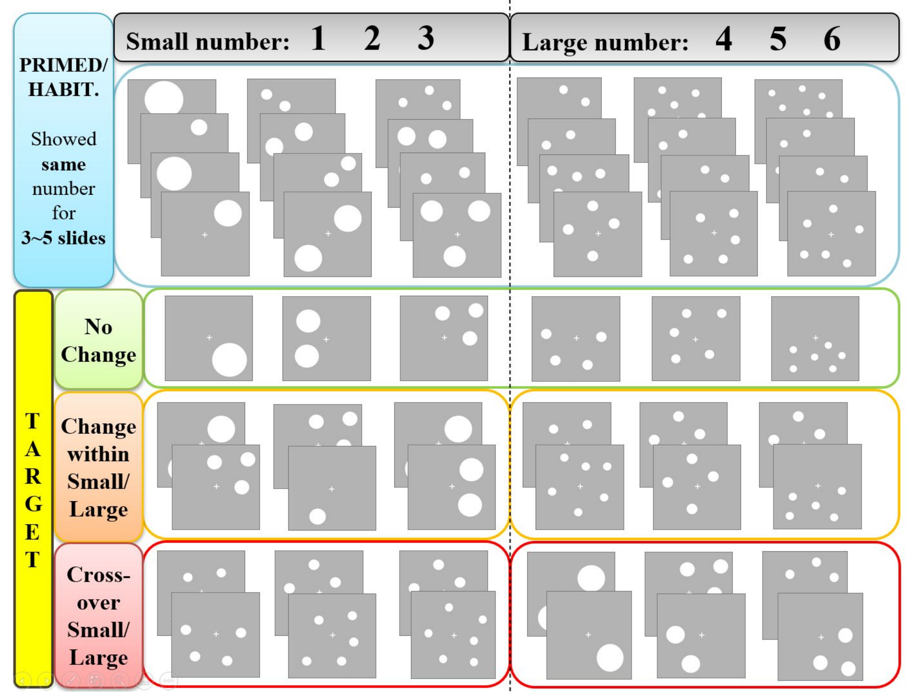

Fig. 1: Current Study Design & Examples of Numerical Stimuli

<u>**Fixing Boredom Problems in Design:**</u>

- Participant boredom was a serious concern when replicating the passive-viewing procedure from Hyde & Spelke (2012).
- To actively engage participants, our study asked participants to detect changes in the numerical value, and to press a key when such changes occurred. Number of trials between changes varied between 3 and 5 so as to be unpredictable.
- Participants were updated on their number of correct responses between testing blocks. At the end of the experiment, each participant was given a lottery scratch off card for every 50 items they got correct.
- Data from both change trials and no change trials were used in the ERP analysis.

<u>**Looking for Categorical Small-Large Differences within a Narrower Range (1 to 6):**</u>

- The Hyde & Spelke (2012) study examined differences between small (1~3) and large (8~24) set sizes that were perceptually distinct with a wide gap between the small and large number ranges.
- Our study examined the small-large distinction within a narrower continuous range of 1 to 6, to see if there was a clear categorical boundary between responses to small and large numbers.
- We wanted to see if there was a distinct ERP response pattern when changes crossed over between small (1~3) and large (4~6) set sizes as compared to responses to within set changes.

Target # / Primed #	1	2	3	4	5	6
1	Same #	inc.SS	inc.SS	inc.SL	N/A	N/A
2	dec.SS	Same #	inc.SS	inc.SL	inc.SL	N/A
3	dec.SS	dec.SS	Same #	inc.SL	inc.SL	inc.SL
4	dec.LS	dec.LS	dec.LS	Same #	inc.LL	inc.LL
5	N/A	dec.LS	dec.LS	dec.LL	Same #	inc.LL
6	N/A	N/A	dec.LS	dec.LL	dec.LL	Same #

Pairs of numerical change (e.g., "1→2" = the Primed # is "1", followed by the Target # as "2")	No change (Same number)	Small → Small (SS)		Large → Large (LL)		Cross-over Small-to-Large (SL) & Large-to-Small (LS)	
		inc.	dec.	inc.	dec.	increase	decrease
	1→1; 2→2; 3→3; 4→4; 5→5; 6→6	1→2; 1→3; 2→3	2→1; 3→1, 3→2	4→5; 4→6; 5→6	5→4; 6→4; 6→5	1→4; 2→4; 2→5; 3→4; 3→5; 3→6;	4→1; 4→2; 4→3; 5→2; 5→3; 6→3

Fig. 2: Pairs of quantities tested: the Primed/Habituated number appears first, followed by the Target number.

Procedure

- 15 right-handed adults (4 males), aged 23 - 43 years (mean = 27.7) participated; Told to press a key whenever they detected a numerical change in the dots.
- EEG data was examined to extract Event Related Potentials (ERPs) associated with Habituation to small (1, 2, 3) and large (4, 5, 6) cardinal values.
- Reaction times, accuracy and ERPs were examined.
- Numerical Change conditions: Within Small, Within Large, Crossover Small-to-Large; Crossover Large-to-Small
- Change Direction variable: Increasing (e.g., 1→2; 5→6) or Decreasing (e.g., 5→4; 3→2)
- Numerical Change distance: Difference of 1, 2, 3 (*see **Fig. 2** for all numerical pairs*)

EEG Acquisition and Processing

- Apparatus: 128-channel EGI Geodesic Sensor Net with High Impedance Amplifier
- EEG Recording
  - Recorded in a shielded sound attenuating chamber
  - Amplified analog voltages were stored digitally
  - The signals were recorded a 0.1- 100 Hz bandpass filtered
  - Sampled and digitized at 250 Hz using Net-Station EEG acquisition software and EGI amplifier
  - Impedance of electrodes was kept below 50 kΩ
  - Individual voltages were referenced to the average across all electrodes
- EEG Data Processing
  - 40 Hz low-pass digital filter applied
  - Segmentation of 800ms length epochs, starting 100 ms before onset of stimuli
  - Artifact rejection
  - Epochs associated with the same category (**i.e., no change, within small, within large, or crossover**) for Decreasing and Increasing change conditions were averaged within subjects
  - EEG recordings were re-referenced to average and the baseline correction was performed to 100ms interval preceding the stimulus onset.
  - Responses were averaged within participants.
- Time windows and ERP components:
  - Time Slice 1 (N170): 125~200 ms
  - Time Slice 2 (P3b): 435~535 ms
- Montages for ERP components (see ***Fig. 3***)
  - Pz area: 62, 78, 77, 72, 67, 61, 54, 55, 79
  - Parietal Occipital Temporal (POT) junction: 66, 65, 59, 60, 67, 71, 70 (Left); 84, 76, 77, 85, 91, 90, 83 (Right)

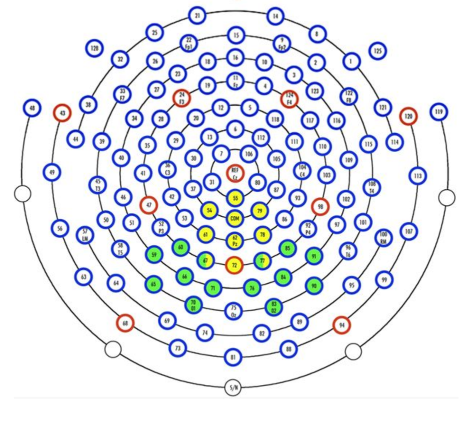

***Fig. 3**: Map of electrode groupings used for averaging and analysis: Pz area (yellow) and POT (green)*

ERPs to Cardinal Numerical Values

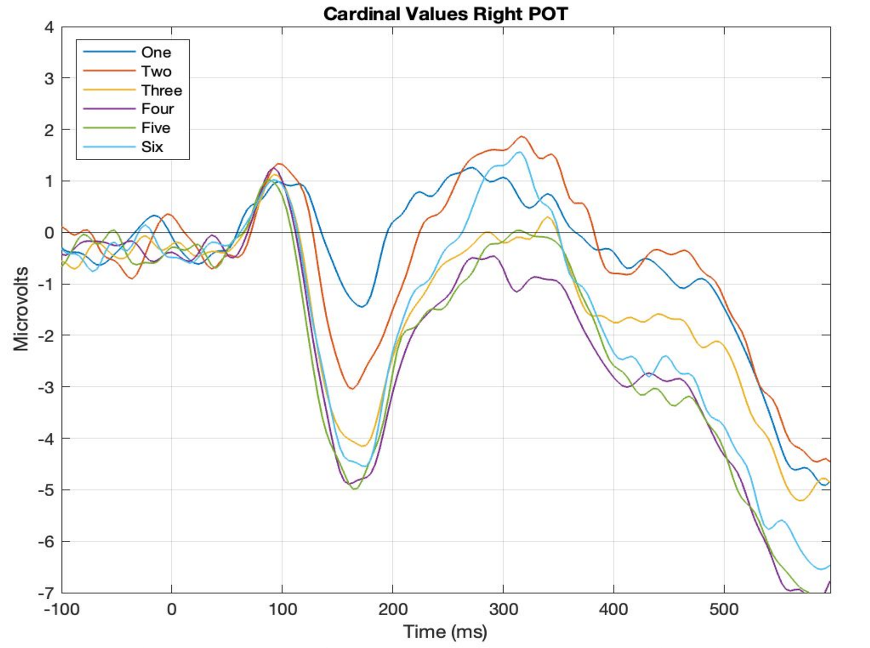

***Fig. 4**: N170 amplitude (microvolts) measured at 125 - 200ms for cardinal magnitudes (1~6) in primed/habituation trials without change in numerical value over the right POT region.*

- N170 (125-200 ms) was measured over the Parietal-Occipital-Temporal (POT) junction (*see Fig. 3*), where there was a separation of ERPs within the subitizing numerical range (1~3), but not beyond (4~6).
- As cardinal value increases, more objects are encoded in early visual working memory, leading to stronger N170 ERP amplitudes.
- Post-hoc comparisons and repeated pair-wise contrasts revealed that the N170 amplitude for One is different from Two; Two is different than Three, and Three is different than the later cardinal values (all p's > 0.005).
- In the large number range (Four and above), the amplitudes were not discernible from each other.
- Stronger amplitudes were observed over the right hemisphere (consistent with Hyde & Spelke, 2012), but there are similar N170 patterns on the left.
- Relative magnitudes of N170 deflections corresponded to ordered numerical magnitudes within the small-number range, but not within the large number range.
- This scaling of the N1 ERP to numerical magnitude replicates Hyde & Spelke (2012).
- Scaling is clearer in our data, and the categorical break between "1" and "2", followed by "3" and 4~6 is apparent.
- ***Fig. 4*** is based on ERP data from only "same number" adaptation trials, not from numerical change trials.

Behavioral Results

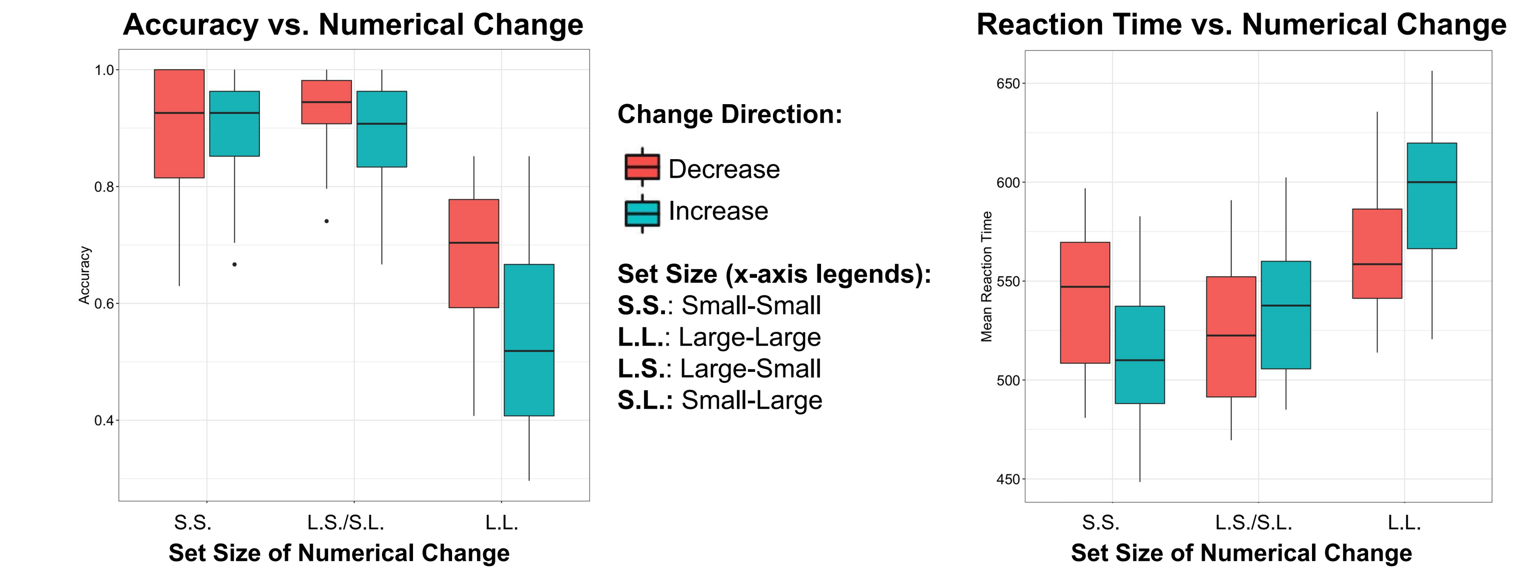

***Fig. 5**: Numerical Change conditions vs. Accuracy (Left plot) and Reaction Time (Right plot)*

- The effects of Size (p<0.0001) and Direction (p<0.001) on Accuracy were significant.
- Accuracy increases as detection of numerical change becomes easier.
- Accuracy is lower for increasing change compared to decreasing change.
- Accuracy was lowest for Large-Large conditions.
- The effects of Size (p<0.0001) and Direction (p<0.01) on Reaction Time were significant. Size and Direction had a significant interaction effect on (p<0.0001).
- Reaction time was longest for Increasing-Large number change (p<0.05), followed by Decreasing-Large.
- This is reflected in their later P3b deflection (see ***Fig. 10***).

Linking Behavior to ERP Data

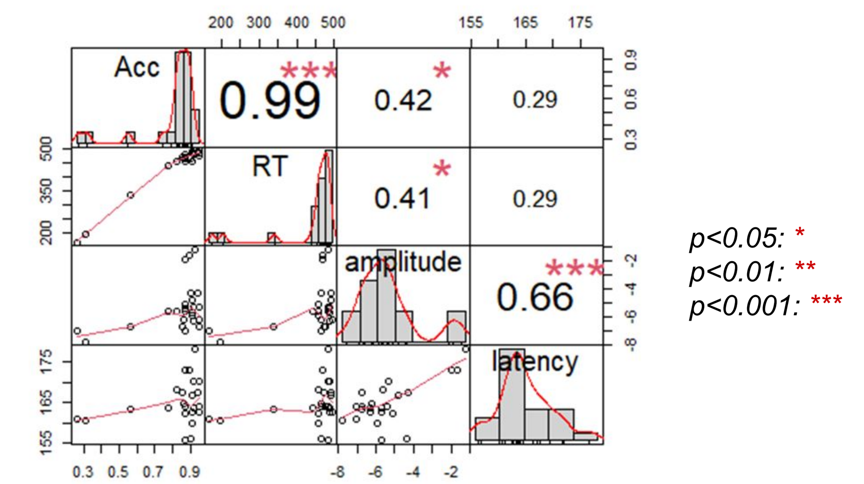

***Fig. 6**: Accuracy vs. Reaction Time vs. N170 Amplitude vs. N170 Latency over right POT, averaged across subjects:*

- Accuracy and RT are very strongly positively correlated with each other.
- Accuracy and RT are moderately correlated with N170 amplitude, and N170 amplitude and latency have a moderate high positive correlation.

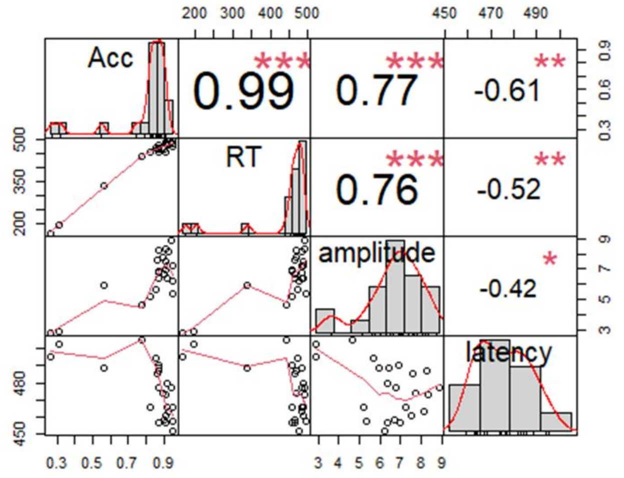

***Fig. 7**: Accuracy vs. Reaction Time vs. P3b Amplitude vs. P3b Latency, averaged across subjects:*

- Accuracy and RT are strongly correlated with Amplitude and Latency of P3b ERP signal.
- Accuracy and RT are negatively correlated with latency and positively correlated with amplitude, and P3b amplitude and latency have a moderate negative correlation.

Context-Updating & Working Memory Model

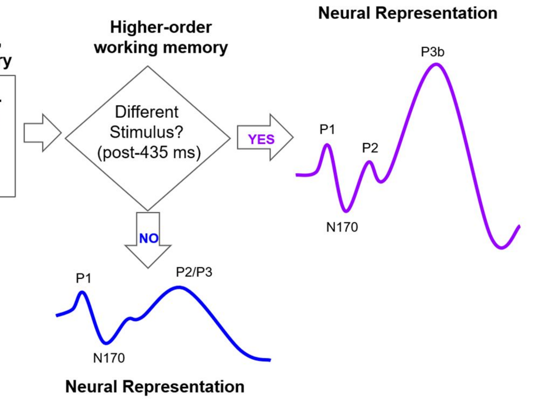

***Fig. 8**: Iconic & Working Memory in Numerical Change: Model adapted from Polich (2007)*

- The context-updating theory of the P3b is related to updating one's working memory in change detection paradigms, where an incoming sensory input is evaluated as being the same or different from the previous context (Polich, 2007).
- If this input is different, it elicits an updating of a given neural representation which is reflected in a P3b deflection at ~400ms.
- We propose that at an earlier sensory stage (~125ms) in numerical context-updating, objects are encoded or off-loaded from iconic memory, which modulates the N170, before integrating this information at later cognitive stages.

Numerical Change: ERP Results

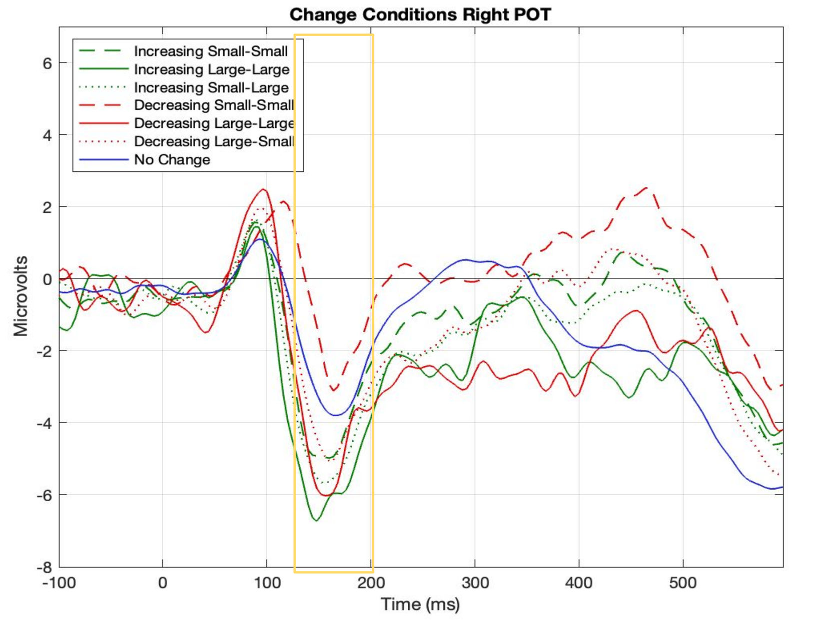

***Fig. 9**: N170 amplitudes for different numerical Change conditions over the right POT area, with "Decrease Small-Small" having the weakest amplitude.*

N170 over the Parietal-Occipital Temporal (POT) area: Early sensory ERP of Numerical Change Detection

- In the POT area, ***Fig. 9*** shows that the weakest N170 amplitude was measured in the "Decrease Small-Small" condition, compared to "No Change", while the rest of the change conditions showed stronger amplitudes.
- Results indicate that at 125-200ms, the POT is "off-loading" objects from visual iconic memory with decreasing small numbers (in the subitizing range), but not for large numbers.
- When cognitive loads are larger and more objects are encoded in iconic working memory, the N170 amplitude is stronger.

P3b over the mid-Parietal (Pz) area: Later cognitive ERP of Numerical Change Detection

- ***Fig. 10*** shows that over the Pz area at 435-535 ms, "Decrease Small-Small" has the strongest P3b amplitude, while Increase and Decrease Large-Large have weaker P3b amplitudes.
- As the number stays the same in No Change, there is a weak P2 signal instead.
- Change within small numbers (1~3) had the highest P3b amplitude, while change within large numbers (4~6) had the lowest amplitude (p<0.01), with no differences based on change direction.
- P3b latency was the highest in Large-Large conditions, followed by the Small-Small conditions, and least for the crossovers (p<0.0001).
- While there were no direct effects for direction of change, we found significant interaction effects (p<0.001). Within the small condition, decreasing trials had higher latency than increasing trials.
- When change detection is easier, it is associated with higher P3b amplitudes, reflecting context-updating processes.

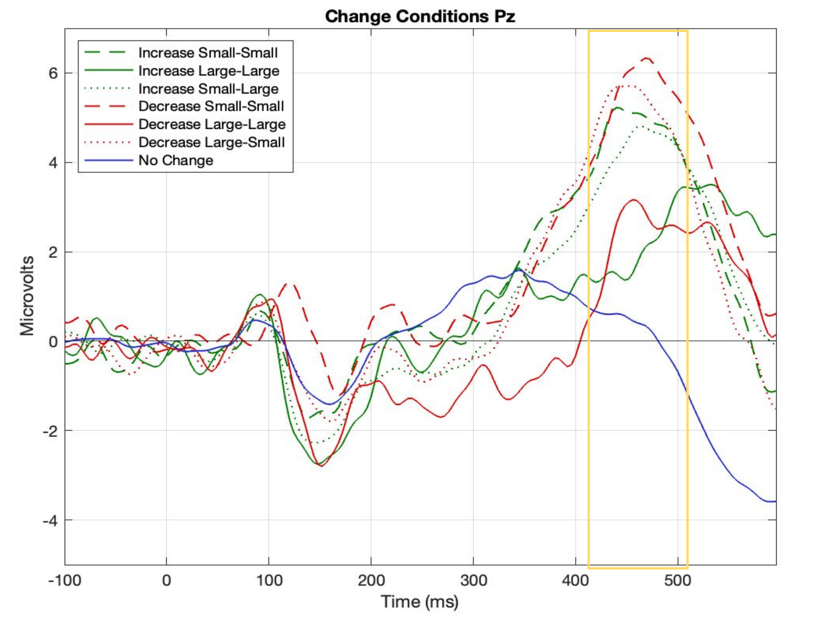

***Fig. 10**: P3 amplitudes for different numerical Change conditions over the mid-Parietal area, with Small and Crossover Sets showing stronger P3b amplitudes; Large Sets showing weaker and later P3b peaks; and No Change conditions showed the earliest weakest P3.*

Conclusions

- Our findings suggest a neural basis for the differentiation of small vs. large number perception at early stages of processing, and a later stage that involves more complex numerical processing that is employed in our numerical change detection task.
- In contrast to Hyde & Spelke (2012), who examined distant small (1, 2, 3) vs. large (8, 16, 24) numbers, we examined a smaller numerical range (1-6), so that small (1-3) vs. large (4-6) contrasts were along a numerical continuum.
- Within this continuous range, we found N170 amplitudes commensurate with cardinal values in the small range (1, 2, 3), but not in the large range (4, 5, 6), where the process of encoding/off-loading objects in memory determines the amplitude strength, suggesting that numbers in the subitizing range are individuated in working memory.
- Distinctions in P3b waveforms also reflect a clear categorical break between increasing vs. decreasing, and small vs. large numbers, where easier/small number change conditions have stronger amplitudes than harder, large number conditions, suggesting more difficulty with updating the context in the latter.
- Overall findings align with the context-updating model (Polich, 2007; see ***Fig. 8***), where working memory representations differ between small and large numbers, as well as increasing and decreasing change.

References

Hyde, D. C., & Spelke, E. S. (2012). Spatiotemporal dynamics of processing nonsymbolic number: An event-related potential source localization study. *Human Brain Mapping, 33*(9), 2189–2203. https://doi.org/10.1002/hbm.21352

Gordon, P. (1994, July). *Innumerate Amazonians and Kronecker's Theism: One-two-many Systems and the Artificialism of Number*. European Society for Philosophy and Psychology, Paris, France.

Polich, J. (2007). Updating P300: An integrative theory of P3a and P3b. *Clinical Neurophy \siology, 118*(10), 2128–2148. https://doi.org/10.1016/j.clinph.2007.04.019

Polich, J. (2011). Neuropsychology of P300. In E. S. Kappenman & S. J. Luck (Eds.), *The Oxford handbook of event-related potential components* (pp. 159–188). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780195374148.001.0001

Temple, E., & Posner, M. I. (1998). Brain mechanisms of quantity are similar in 5-year-old children and adults. *Proceedings of the National Academy of Sciences, U.S.A.*, *95*(13), 7836–7841.